学生诚信协议：答题期间，我确定没有就所涉及的问题或结论，与任何人、用任何方式交流或讨论，我确定我所填写的答案均为我个人独立完成的成果，否则愿接受本次成绩无效的处罚。

选择题：每小题 5 分，答对加 5 分，答错不扣分，共 200 分。

1. The mixed number $2\frac{1}{4}$ is equivalent to many improper fractions that have integer numerators and denominators. The numerator of such a fraction could be any of the following except

   A) 24 B) 27 C) 36 D) 45

2. At my store, $1 of every $5 in sales is profit. If I split 10% of all profits equally among 10 people, each gets __?__% of the total sales.

   A) 0.2 B) 2 C) 5 D) 20

3. $2^{24} =$ __?__ $\times 2^{12}$

   A) 2 B) 2^2 C) 2^{12} D) 2^{36}

4. Of the following, which expression has the least value?

   A) $\dfrac{3^{100}}{4}$ B) $\left(\dfrac{3}{4}\right)^{100}$ C) $\dfrac{3}{4}$ D) $\dfrac{3}{4^{100}}$

5. I randomly select a positive integer less than 100. The probability that it is the product of exactly 3 different primes is

   A) $\dfrac{1}{99}$ B) $\dfrac{4}{99}$ C) $\dfrac{5}{99}$ D) $\dfrac{8}{99}$

6. If the average of 3 consecutive ticket numbers is odd, then the sum of the least and greatest ticket numbers could be

   A) 18 B) 20 C) 24 D) 28

7. Eve counted to 4^{60} by consecutive powers of 2, starting with $2^1, 2^2, 2^3, \ldots$. How many powers of 2 did Eve count?

   A) 30 B) 120 C) 240 D) 3600

8. How many even integers between 1 and 1 000 000 have digits that are all primes?

   A) 1365 B) 3906 C) 5400 D) 19 530

9. If 6 identical machines can fill 80 bottles of soda in 12 seconds, how many seconds would it take 36 of the same machines to fill 240 bottles of soda?

   A) 6 B) 12 C) 18 D) 24

10. Of my 100 favorite released songs, 42% were released after the year 2015 and 76% were released before the year 2017. What percent of my favorite songs were released in 2016?

    A) 18%   B) 24%   C) 34%   D) 58%

11. (The number of positive even integers less than 10^6 that are perfect squares) : (the number of positive odd integers less than 10^6 that are perfect squares) =

   A) 1:1   B) 2:1   C) 499:500   D) 999:1000

12. Of the following, which is a multiple of 4?

    A) $2017^{2018} + 1$ B) $2017^{2018} + 3$ C) $2017^{2018} + 5$ D) $2018^{2017} + 1$

13. If the sum of the measures of two angles of a parallelogram is 108 degrees, the sum of the measures of three of its angles could be

    A) 72 degrees   B) 162 degrees   C) 234 degrees   D) 252 degrees

14. A rectangle is partitioned into 9 different squares, as shown at the right. The area of the smallest square, shown fully darkened, is 1. Two other squares have areas of 196 and 324, as shown. What is the area of the shaded square?
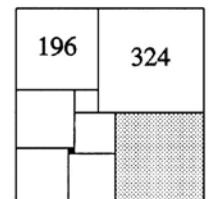
    A) 196   B) 198   C) 223   D) 225

15. In a regular 10-sided polygon, two pairs of different vertices (four different vertices altogether) are chosen at random, so that all points chosen are distinct from each other. What is the probability that the line segments determined by each pair of points do *not* intersect?

    A) 1/3   B) 2/3   C) 4/9   D) 5/16

16. Mom, Dad, and I each write a positive integer. My number is least and Dad's is greatest. The average of all 3 numbers is 20. The average of the 2 smallest numbers is 8. If Dad's number is d and if my number is m, what is the greatest possible value of $d - m$?

    A) 42   B) 43   C) 44   D) 45

17. When the square of an eight-digit integer is subtracted from the square of a different eight-digit integer, the difference will sometimes have eight identical even digits. What are both possible values of the repeated digit in such a situation?

    A) 4, 8   B) 2, 4   C) 6, 8   D) 2, 6

18. A line segment is drawn from the upper right vertex of a parallelogram, as shown, dividing the opposite side into segments with lengths in a 2:1 ratio. If the area of the parallelogram is 90, what is the area of the shaded region?
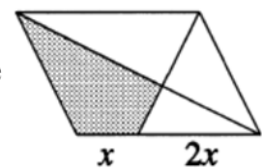
    A) 30   B) 31   C) 32   D) 33

19. If 8 different integers are chosen at random from the first 15 positive integers, what is the probability that an additional number chosen at random from the remaining 7 positive integers is smaller than every one of the 8 originally chosen positive integers?

    A) 1/13   B) 23/225   C) 13/125   D) 1/9

20. If the perimeter of an isosceles triangle with integral sides is 2017, how many different lengths are possible for the legs?

    A) 500   B) 502   C) 504   D) 506

21. A hexagon is inscribed in a circle as shown. If lengths of three sides of the hexagon are each 1 and the lengths of the other three sides are each 2, what is the area of this hexagon (rounded to the nearest tenth)?

    A) 5.2   B) 5.6   C) 6.0   D) 6.4

22. As shown, a small square is inscribed in one of the triangles formed when both diagonals of a larger square are drawn. If the area of the larger square is 144, what is the area of the smaller square?
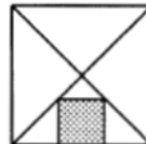
    A) 12   B) 14   C) 16   D) 18

23. What is the number of different ordered triples of positive primes (p, q, r) that satisfy $p^q + 1 = r$?

    A) 0   B) 1   C) 2   D) 3

24. What is the number of different ordered pairs of integers (x,y) that satisfy $5x^3 + 2xy - 23 = 0$?

    A) 3   B) 4   C) 5   D) 6

25. Trisection points on opposite sides of a rectangle are joined, as shown. If the area of the shaded region is 2018, what is the area of the rectangle?

    A) 8072   B) 10090   C) 12108   D) 14126

26. Three beavers (one not shown) take turns biting a tree until it falls. The second beaver is twice as likely as the first to make the tree fall. The third is twice as likely as the second to make the tree fall. What is the probability that a bite taken by the third beaver causes the tree to fall?

    A) 1/3   B) 1/2   C) 3/7   D) 4/7

27. If two altitudes of a triangle have lengths 10 and 15, what is the smallest integer that could be the length of the third altitude?

    A) 5   B) 6   C) 7   D) 8

28. A man had two horses. He sold one of them on Tuesday for $198 and made a profit of ten percent. On Wednesday, he sold the other one for $198 and took a loss of ten percent. Tallying up his two deals, did he show a net profit or a loss?

    A) Even   B) A net profit of $6

    C) A net profit of $4   D) A net loss of $4

29. What is the ratio, larger to smaller, of a rectangle's dimensions if half of the rectangle is similar to the original rectangle, as shown?
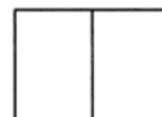
    A) 2   B) 1.5   C) $\sqrt{2}$ D) $\sqrt{3}$

30. The sum of Jane's and John's ages is 22. However, Jane will be seven times as old as John is now when she becomes twice his age. How old is Jane?

    A) 20   B) 15   C) 12   D) None of the above

31. A landscaping job takes a certain number of men as many days to complete as there are men working. If the addition of six men would complete the job in just one day, how many men were working?

    A) 1   B) 2   C) 3   D) 6

32. If Mary is twice as old as Ann was when Mary was as old as Ann is now, and Mary is 32, how old is Ann?

    A) 20   B) 24   C) 32   D) None of the above

33. Two circles intersect at (6, 3). Each circle is tangent to both coordinate axes. Find the sum of radii of the two circles.

    A) 12   B) 15   C) 16   D) None of the above

34. If a stick is broken at random into three pieces (that is to select, independently and randomly, two points from the points that range uniformly along the stick, then break the stick at these two points), what is the probability that the pieces can be put together into a triangle?

A) 1/4   B) 1/3   C) 1/2   D) 3/4

35. What is the number of different ordered triples of positive integers (a, b, c) that satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

and $a \geq b \geq c$.

A) 2   B) 3   C) 4   D) 6

36. Function $F(k)$ is defined for positive integers as $F(1) = 1$, $F(2) = 1$, $F(3) = -1$, and $F(2k) = F(k)$, $F(2k + 1) = F(k)$, for all $k \geq 2$. Find the value of the sum $F(1) + F(2) + F(3) + ... + F(2016) + F(2017)$.

A) 25   B) 30   C) 36   D) None of the above

37. In Planet Utopia, each person has Green, Blue, Black, or Purple eyes. A pair of genes decides the eye color, for which gene order doesn't matter, Gene has three types: Green, Blue, or Black. The table on the right decides Gene vs Eye color.
A couple contributes one gene each to their offspring. If one has Blue eyes and the other has Black eyes, what is the probability that their offspring has Purple eyes?

Gene \ Gene	Green	Blue	Black
Green	Green	Blue	Black
Blue	Blue	Blue	Purple
Black	Black	Purple	Black

A) 3/8   B) 7/16   C) 5/8   D) None of the above

38. In the figure on the right, how many different paths are there from $A \rightarrow D$ without going through the same node twice? Skipping a node or more is an acceptable path.
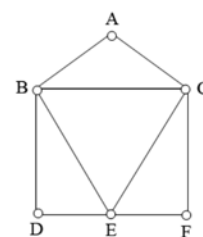
A) 6   B) 8   C) 10   D) 12

39. Find the remainder when the monomial x^{2016} is divided by $(x^2 + 1)(x^2 + x + 1)$.

A) 1   B) x C) $2x^2$ D) x^3

40. Let a, b, c be distinct nonzero real numbers, such that $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a} = k$. Compute the value of abc.

A) k B) $-k$ C) $2k^2$ D) $2k^3$

2017-2018 年度美国"大联盟"(Math League)思维探索第一阶段活动

八、九年级试卷答案

题号	1	2	3	4	5	6	7	8	9	10
答案	A	A	C	D	C	A	B	A	A	A
题号	11	12	13	14	15	16	17	18	19	20
答案	C	B	C	D	B	B	A	D	D	C
题号	21	22	23	24	25	26	27	28	29	30
答案	B	C	B	B	C	D	C	D	C	D
题号	31	32	33	34	35	36	37	38	39	40
答案	C	B	D	A	B	D	D	C	A	B